LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **NOVEMBER 2023**

PST 3502 – STOCHASTIC PROCESSES

Date: 01-11-2023 Dept. No. Max. : 100 Marks
Time: 01:00 PM - 04:00 PM

SECTION –A

Answer ALL the questions . **10 x 2 = 20 Marks**

1.Define index parameter and state space for a Markov process.

2.Write two properties of periodicity.

3.When a state is said to be (i) positive recurrent and (ii) null recurrent ?

4.State Abel lemma.

5.Show that recurrence is a class property.

6.Write the postulates of a birth and death process.

7.Explain the counting process.

8.Define sub-martingale for Markov process.

9.Provide any two examples for branching process.

10. Define covariance stationary process.

SECTION –B

Answer any FIVE questions . **5 x 8 = 40 Marks**

11. Prove that one-dimensional random walk is recurrent.

12. Let a Markov chain have four states 1,2,3 and 4 with the following one-step transition probabilities: $P_{12} = P_{13} = P_{14} = 1/3$,
$P_{21} = P_{22} = P_{23} = 1/3$, $P_{32} = P_{34} = 1/2$ and $P_{41} = 1$. Find the stationary distribution.

13. State and prove the theorem used to find the stationary probability distribution when the Markov chain is positive recurrent, irreducible and aperiodic.

14. For the Yule process under the condition that X(0) = N =1 obtain mean and variance.

15. Explain Type II counter model in renewal process with the necessary diagram.

16. (a) Let $Y_0 = 0$ and $Y_1, Y_2, \ldots$ be independent and identically distributed random variables with mean 0 and variance σ^2. If $X_0 = 0$ and $X_n = (Y_1 + Y_2 + \ldots + Y_n)^2 - n\,\sigma^2$ show that $\{X_n\}$ is a martingale with respect to $\{Y_n\}$. (5)

(b) Explain Doob's martingale process. (3)

17. If m denotes the average number of offspring per individual and π the probability of extinction then show that $\pi = 1$ if $m \leq 1$ and $0 < \pi < 1$ if $m > 1$.

18. Explain two contrasting stationary processes.

SECTION –C

Answer any TWO questions . **2 x 20 = 40 Marks**

19. (a) Derive the differential equations for a pure birth process by clearly stating the assumptions. (8)

(b) State and prove the basic limit theorem of Markov chains. (12)

20. Consider the state space S = {1,2,3,4,5,6} with the one-step transition probabilities: $P_{11} = 1/3$, $P_{13} = 2/3$, $P_{22} = 1/2$, $P_{23} = P_{25} = 1/4$, $P_{31} = 2/5$, $P_{33} = 3/5$, $P_{42} = P_{43} = P_{44} = P_{46} = 1/4$, $P_{55} = P_{56} = 1/2$, $P_{65} = 1/4$ and $P_{66} = 3/4$.

(a) Draw the transition diagram and form the transition matrix. (2)

(b) Find the equivalence classes.(2)

(c) Determine period of states. (2)

(d) Show that states 1,3,5 and 6 are recurrent.(10)

(e) Prove that states 2 and 4 are transient.(4)

21. (a) Show that Poisson process can be viewed as a renewal process. (10)

(b) State and prove the elementary renewal theorem. (10)

22. (a) Derive mean and variance for branching process. (10)

(b) Derive M(t) for a linear growth process with immigration. (10)

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